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Horizontal diffusion in the atmosphere in the light of air trajectories.

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C. S. Durst, A. F. Crossley and N. E. Davis

(Meteorological Office)

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Horizontal diffusion in the atmosphere in the light of air trajectories

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Summary

The dispersion of the end points of geostrophic trajectories is examined for a considerable number of cases. The standard vector deviations of the end points of these trajectories are computed after various times, and also correlation coefficients are calculated between the initial wind velocities and the velocities at later times following the trajectories. An attempt is made to determine an empirical law connecting the standard vector deviation of the end points with the time during which the trajectories had been pursued. There is good agreement with a law based on an exponential value for the Lagrangian correlation coefficient of wind with time.

Introduction

If a trajectory of the free air is drawn from a point it will follow a meandering path; if at subsequent times a second, third and successive trajectories are initiated from the same point at equal time intervals, their paths will form a plume flowing out in the direction of the general wind stream, as is shown in Figures 1a and 1b. These plumes are not of exactly the same type as for example the plume of smoke from a chimney emitting a continuous source of pollution but are more closely parallelled by the pattern on the ground of the pollution deposited from such a chimney. The analogy is not exact in the case of geostrophic trajectories because in a smoke plume the individual particles are free to move vertically as well as horizontally. A geostrophic trajectory is the quasi-horizontal path of a point element, but the cross-section of a smoke plume is affected by the eddies within itself and in its surroundings. Sutton (1) emphasised this feature of smoke and in his hypothesis he has suggested that the dimensions of the eddies which operate on the smoke are essentially related to the distance which the smoke has travelled from its source. Thus it may be that there is a difference between the plume of smoke and the bundle of trajectories but such studies as are given in this paper may it is hoped enable the fundamental characteristics of large scale turbulence to be more clearly understood. For that reason we have included the summarised data concerning the trajectories in an appendix so that they may be available to other workers.

To discuss these plumes of trajectories we should follow the true paths of selected elements of air, but since it is not in general possible to do so on the scale of synoptic charts, there have been substituted instead "geostrophic trajectories". These are formed, according to common practice, by measuring the geostrophic wind near the point of origin on a synoptic or pressure contour chart, plotting the position which the air at that point will have reached after half the time interval to the next chart, measuring the geostrophic wind on the next chart, at the position at which the air will probably have arrived at the time of that (second) chart, drawing the probable track of the air up to the mid time between the second and third charts and so on. There are errors inherent in this process which do not necessarily cancel out but nevertheless when a large number of trajectories are drawn it is believed that the technique serves to produce some important data concerning the atmosphere.

The purpose of this paper is to discuss the distribution of the end points of air trajectories, initiated at regular intervals (i) at a point above the earth's surface and pursued for a given length of time (t). It is clear that the distribution is related not only to the time during which the trajectories are pursued but also to the length of the period (T) over which they are regularly initiated. Thus there are three time scales, but if N is the number of trajectories followed $(N-1)\,i = T$ and provided N is reasonably large the plume can be considered fairly representative of the wind régime and so the magnitude of i does not appear to be significant to the same extent as that of t and T. It is clear that the value of T is of importance in relation to the general pattern of the wind régime prevailing during the period when the trajectories are being drawn. This is at once apparent in the comparison of Figures 1a and 1b which are drawn at 700 mb over N. America where data for a large area are most complete. In the former case the general circulation was strong and uniform, in the latter there was much greater meandering of the trajectories and greater variability of the individual winds. The effect is not necessarily seasonal but it can readily be seen to be dependent on the frequency and type of eddies (depressions and anticyclones) which were passing during the period. For instance, if numerous centres of disturbances were passing over the neighbourhood of the initiation point the plume would have greater spread than if the centres of depressions were following a track to the north. Another pair of plumes is shown in Figures 2 and 3 also initiated at 700 mb. In these cases the date of the individual trajectories is indicated by a figure at the end.

General theory

Let us suppose that a large number (N) of trajectories be drawn at regular intervals of time T/N from a particular point in the earth's atmosphere. After an interval of t hours a typical trajectory has extended to the point x_t, x_t being the vector from the point of origin to the end point of the trajectory as shown in Figure 4. Let V be the vector wind at any point on the trajectory. Then by the simple summation of vectors

$$x_t = \int_o^t V\,dt \tag{1}$$

Further, if we sum over the N trajectories we get

$$\frac{\sum x_t}{N} = \frac{1}{N}\int_o^t \sum V\,dt \tag{2}$$

which may be written $$[x_t]_T = \int_o^t [V]_T\,dt \tag{3}$$

the brackets indicating that a mean over the period T is taken and accordingly

$$x_t - [x_t]_T = \int_o^t \{V - [V]_T\}\,dt = \int_o^t v_T\,dt \tag{4}$$

where v_T is the vector departure from the mean taken over the period T. That is, the vector departure from mean of the trajectory ends can be obtained from the integral with time of the vector departure from means of the wind. Moreover if S is the standard vector deviation of the end point of the trajectories at time t after initiation

$$S^2 \equiv \overline{\{x_t - [x_t]_T\}^2} = \int_o^t \{v(a)\}\,da \int_o^t \{v(b)\}\,db \tag{5}$$

If we define σ_T to be the standard vector deviation of wind in the period T, i.e.,

$$\sigma_T^2 = \overline{\{v(a)\}^2} = \overline{\{v(b)\}^2} \tag{6}$$

equation (5) gives

$$S^2 = \sigma_T^2 \int_0^t da \int_0^t r_T(a-b)\, db \tag{7}$$

where $0 \leq a \leq t$, $0 \leq b \leq t$ and $r_T(a-b)$ is the stretch vector correlation coefficient* between $v(b)$ and $v(a)$. But $r_T(a-b)$ is an even function so if we consider only $a \geq b$ and put $a-b=\xi$, $0 \leq \xi \leq a$, then the contribution to the double integral from $a \geq b$ is

$$\sigma_T^2 \int_0^t da \int_0^a r_T(\xi)\, d\xi \tag{8}$$

and there is an equal contribution from $a < b$ whence

$$S^2 = 2\sigma_T^2 \int_0^t \int_0^a r(\xi)\, d\xi\, da \tag{9}$$

Differentiating twice we see that

$$r_T(t) = \frac{1}{2\sigma_T^2} \frac{d^2S^2}{dt^2} \tag{10}$$

Expression (9) is of the same type as that given by Taylor (2).
Now

$$\overline{\{V(a)-V(b)\}^2_T} = \overline{\{v(a)-v(b)\}^2_T} = \overline{v(a)^2_T} + \overline{v(b)^2_T} - 2\overline{v(a)_T \cdot v(b)_T} \tag{11}$$

$$= 2\sigma_T^2 - 2\sigma_T^2 r_T \tag{12}$$

and from equations (9) and (12) we can say

$$S^2 \equiv \overline{\{x_t - [x_t]\}^2} = \sigma_T^2 t^2 - \int_0^t \int_0^b \overline{\{V(a)-V(b)\}^2}\, da\, db \tag{13}$$

The form of $r(t)$

It was pointed out by Sutton (1) (p.284) that for motion near a smooth boundary dimensional considerations require

$$r(t) = f\left(\frac{\overline{\omega}^2 t}{V}\right) \tag{14}$$

/where

* For the definition of the "stretch vector correlation coefficient" vide Geophysical Memoir 93 or Prof. Notes 121.

where ν is the kinematic viscosity and $\overline{\omega^2}$ the mean square eddy velocity. For large scale horizontal diffusion a similar argument would lead to

$$\tau(t) = f\left(\frac{\sigma^2 t}{K}\right) \tag{15}$$

where σ is the standard vector deviation of wind and K the eddy viscosity for lateral mixing.

Sutton proposed the expression

$$\tau(t) = \left(\frac{\nu}{\nu + \overline{\omega^2} t}\right)^n \tag{16}$$

where n is a numerical constant between 0 and 1. When the boundary is dynamically rough, the kinematic viscosity may be replaced by the macro-viscosity $N = u_* z_0$ where u_* is the friction-viscosity and z_0 the roughness-length (Sutton p.82).

Thus N has no direct counterpart in the free atmosphere. Sutton pointed out that provided ν in equation (16) could be ignored in relation to $\overline{\omega^2} t$, S in equation (9) had the form of a power law. The success of this treatment was attributed to the power law taking account of the effects of larger and larger eddies, the further and further the observational position receded from the source of smoke in Sutton's experiments.

Now for values of t greater than, say, t^* large scale diffusion in the free air is controlled by the largest eddies, which in middle latitudes consist of travelling cyclones and anticyclones, and the general long wave pattern, and which are not restricted in size in the way that eddies are restricted by the proximity of a physical boundary. The eddies in the free atmosphere that are concerned in any long period of diffusion may indeed be of various sizes, but their average size does not change with time in any systematic way. Hence $\tau(t)$ might well approximate to a simple exponential function

$$\tau(t) = \exp(-\alpha t), \quad t \text{ greater than } t^* \tag{17}$$

Taylor and others have suggested the exponential law in the past but we infer from the previous argument that the coefficient α has dimensions of σ^2/K. In the sequel this form is taken as

$$\tau(t) = \exp\left(-\frac{\sigma^2 t}{K}\right) \tag{18}$$

which is shown to be physically reasonable, although there may be a suppressed non-dimensional factor of order unity in the index.

It is to be noted that if in expression (16) we put K the eddy viscosity for lateral mixing in place of ν and σ the standard vector deviation of wind in place of $\overline{\omega^2}$, K cannot be ignored in relation to $\sigma^2 t$ unless t is of the order of days. Hence it cannot be argued by a precisely similar argument to that put forward by Sutton that $\tau(t)$ should obey a power law for diffusion on the scale discussed in this paper. Nevertheless a large amount of work has been done on fitting a power law to the data obtained from trajectories and a considerable success has been obtained to which reference is made at the end of the present paper.

It is important to realise that for testing the form of $\tau(t)$ measurements must be made within ranges which are rather strictly confined. It has been pointed out by Taylor that when t is small and $\tau(t)$ is nearly unity then S is initially nearly $\sigma_1 t$; also that if $\tau(t)$ is zero for all $t \geq t_1$, $\int_0^t \tau(\xi)\, d\xi$ tends to a finite limit as t increases and consequently S varies as $\sqrt{t}$ when t is much greater than t_1, no matter what the form of $\tau(t)$

is for values of t less than t_1. More particularly we have for t greater than t_1, $S^2(t) = k(t-t_1) + S^2(t_1)$ where k is a constant. This gives a simple formula for diffusion for times exceeding t_1 and incidentally shows clearly that if we wish to examine the form of $\gamma(t)$, it will not help if we extend the time during which the samples are followed much beyond this critical time t_1.

In the case of geostrophic trajectories in the free air the critical time would appear to be about 40 hours but for the surface trajectories about 18 hours. Consequently in the discussions of the form of $\gamma(t)$ we have not made use of data for trajectories beyond 36 and 18 hours respectively.

However, if we assume that $\gamma(t)$ is exponential in form we can calculate the dispersion of the end points of the trajectories after they have been followed for a given time t by use of (9) which on integration gives

$$\frac{S}{\sigma_T} = \frac{\sqrt{2}}{\alpha}\sqrt{\alpha t - 1 + e^{-\alpha t}} \qquad (19)$$

and the values of S can then be compared with data such as those given in Appendix I.

It is to be noted that for times somewhat greater than t_1, which make $e^{-\alpha t}$ small but for which αt is not great in comparison with unity, there is a range in which S^2 is approximately equal to $2\sigma_T^2\left(\frac{t}{\alpha} - \frac{1}{\alpha^2}\right)$ and so given values of S_1 and S_2 at times t_1 and t_2, both of which are greater than the critical value mentioned above, we can obtain the value of α from the formula

$$\alpha = 2\sigma_T^2\,\frac{t_2 - t_1}{S_2^2 - S_1^2} \qquad (20)$$

or since $\alpha = \frac{\sigma_T^2}{K}$ we can obtain K in the form

$$K = \frac{\sigma_T^2}{\alpha} = \frac{S_2^2 - S_1^2}{2(t_2 - t_1)} \qquad (21)$$

Here K is the eddy viscosity for lateral mixing and is seen to be obtainable directly from the spread of the end points of the trajectories, no matter what the standard deviation of the wind, provided always that the assumption of an exponential form for γ is correct.

Magnitude of errors involved in using geostrophic trajectories

Before proceeding to the discussion of the results a word must be said as to the errors likely to arise in taking the geostrophic trajectories as representative of the true motion of the atmosphere. The principal causes of error are (a) inaccuracy in measurement of geostrophic winds and (b) the inherent unrepresentativeness of the geostrophic wind as a picture of the true air flow. The magnitude of these errors over the British Isles is given by Murray (11) and a note on their effects on trajectories by Durst and Davis (12).

Let the true wind be $V + v$ and the error be ϵ then γ'_T the erroneous correlation coefficient between $v(a)$ and $v(b)$ becomes $\frac{\{v(a)+\epsilon_1\}\{v(b)+\epsilon_2\}}{\sigma_T^2 + \overline{\epsilon^2}}$

/instead

instead of the true value $\frac{\overline{v(a).v(b)}}{\sigma_T^2}$. Moreover, if we can assume there is little or no correlation between ϵ_1 and ϵ_2 or between ϵ_1 and $v(a)\ldots\ldots$, except possibly very near time $t = 0$,

$$r_T' = \frac{\overline{v(a).v(b)}}{\sigma_T^2 + \overline{\epsilon^2}} \qquad (22)$$

If $\overline{\epsilon^2}$ is small in comparison with σ_T^2 we get

$$r_T - r_T' \doteqdot \frac{\overline{\epsilon^2}}{\sigma_T^2} r_T \qquad (23)$$

If we go back to equation (9) and use the value of σ_T^2 deduced from the actual measurements of velocities (i.e., if we really use $\sigma_T^2 + \overline{\epsilon^2}$) we see that $\sigma_T^2 + \overline{\epsilon^2}$ occurs in fact in both numerator and denominator and in consequence there is no consistent error in S due to errors in the measurements of velocities.

It is true that a large value of ϵ_1 will lead to the apparent trajectory running away from the true trajectory with in consequence a large value of ϵ_2. This effect is discussed by Durst and Davis (12). After 36 hours the results of this effect might produce an error of 250 to 350 km for individual trajectories at 700 mb and when one looks at the magnitude of the standard deviation of end points given in Appendix I it is clear that an error of such an amount may on some occasions produce an effect which is an appreciable proportion of the true S. However, on the whole the error involved in using geostrophic trajectories in determining the values of S is probably not more than 10 per cent at any rate for trajectories extending to 36 or 48 hours.

Empirical values of the correlation coefficient $r(t)$

The values of $r(t)$ have been evaluated from two types of data, (a) from a series of constant level balloons which were flown at 300 mb for periods of 2, 3 or 4 days, the results of which have been published (3) in America, (b) from a considerable series of geostrophic trajectories at levels of 300, 500, 700 mb and some from surface charts. The values from the constant level balloons are set out in Table I, those of the geostrophic trajectories in Table II.

Table I. Values of $r(t)$ derived from constant level balloons flown at 300 mb over America and the North Atlantic in latitudes north of 40°N.

Time interval (hours) (t)		2	4	6	8	10	12	14
Number of pairs of observations		59	51	43	35	31	27	24
Standard differences	N-S	19	22	28	30	33	38	39
	E-W	17	19	22	25	28	30	32
	Vector	26	29	35	39	43	49	50
r (t)		0.72	0.66	0.50	0.38	0.24	0.02	-0.02
α x 10^5 sec^{-1}		4.5	2.9	3.2	3.4	3.9	?	?

In Table I the value of $r(t)$ has been calculated from the mean square differences between observed velocities at a different time interval by means of equation (12), the standard vector deviation being 35 kt. By proceeding in this way it was possible to obtain a considerable number of comparisons from a limited number of trajectories. The correlation coefficients nearly follow an exponential law and values of α have been calculated on the assumption that $r(t)$ is of the form $e^{-\alpha t}$. The mean value of α is $3.6 \times 10^{-5} sec^{-1}$ (or $0.13\ hr^{-1}$). If α is $\frac{\sigma^2}{K}$ we find the value of K to be 9.3×10^{10} in c.g.s. units.

There is however a difficulty in that there are errors in the positioning of the balloons which are stated to range from 20 miles in the case of good fixes to 40 miles in the case of moderately good fixes and up to 60 miles in the case of poor fixes. From internal evidence, however, it is clear that these figures are excessively high, and although any correction for position error will tend to increase all the correlation coefficients, the resulting increase in the value of K would be hardly significant.

Secondly we can use data derived from geostrophic trajectories. In Appendix I there are set out the standard vector deviations of the end points after the trajectories had been followed for various times and also the vector mean distance that the end points had reached during those times.

Further, for a number of groups of trajectories the winds have been tabulated after each time interval and the actual correlation coefficients between the velocity at one time and that at a subsequent time on the same trajectory have been calculated. In Table II there are given the correlation coefficients between measured geostrophic winds at one time and measured geostrophic wind at the appropriate point on the same trajectory at a subsequent time. The number of trajectories on which they are based is given but on each trajectory a number of pairs of points can be taken with the same time interval so that the correlation coefficients are based on several times as many pairs as the number of trajectories, the number of pairs being entered in brackets in the table. Correlation coefficients are given separately for the south-north direction (S) and the east-west direction (W) and a vector correlation coefficient is also calculated and entered below the others. In October and November, 1949 each month was split up into six periods of five days and departures from 5-day means were formed so that the average correlation coefficients were calculated for 5-day periods of initiation. Some specimens of the relationship between $\log_{10} r(t)$ and t are shown in Fig. 5.

The mean value of the logarithms of the correlation coefficients for 12 hours in Table II is -0.238, for 24 hours is -0.464, and for 36 hours is -0.703; the mean value of the standard vector deviation is 18 kt. These values of $\log_{10} r(t)$ have been plotted in Fig. 6 and fit a relationship such that

$$r(t) = e^{-\alpha t}$$

where $\alpha = 1.25 \times 10^{-5}$ sec^{-1} or 0.045 hr^{-1}

If then $\alpha = \frac{\sigma^2}{K}$ and $\sigma = 18$ kt,

$$K = 6.8 \times 10^{10} \text{ cm}^2/\text{sec}.$$

During 1943 to 1946 a very close network of stations was maintained over the British Isles and observations were plotted at 3-hour intervals. Surface geostrophic trajectories were drawn from two points over the British Isles on eight days chosen at random. These trajectories were pursued for 24 hours and new ones initiated every three hours (nine times) so that the period of initiation of each family of trajectories was 27 hours. The winds were measured every three hours and correlation coefficients were calculated for 6, 12, 18 and 24-hour intervals from the 18 comparisons for each family. The results are set out in Table III.

/Table III

Table III. Vector correlation coefficients from surface geostrophic trajectories over the British Isles

Date	Number of trajectories	Standard vector deviation (kt)	Time interval (hours) 6	12	18	24
21/22 Sept., 1943	18	11.3	0.27	0.04	0.02	-0.06
25/26 Sept., 1943	18	7.7	0.43	0.59	0.10	0.21
3/4 March, 1944	18	8.7	0.18	-0.08	0.01	0.13
6/7 March, 1944	18	6.9	0.16	0.02	0.18	0.01
16/17 June, 1944	18	12.9	0.56	0.14	0.07	-0.33
22/23 June, 1944	18	6.9	0.36	-0.29	-0.59	-0.55
12/13 Dec., 1946	18	4.9	0.19	-0.04	0.17	-0.16
16/17 Dec., 1946	18	8.7	0.09	0.07	0.35	0.33
Mean		8.5	0.28	0.06	0.04	-0.05
α $\{$ or $-\frac{1}{t}\log_e r(t)\}$ $\times 10^5$ sec^{-1}			6.1	6.4	5.0	

from which $K = 3.3 \times 10^9$ cm^2/sec.

There seems then to be fairly strong evidence that the correlation coefficients between the winds at the end points of the trajectories follow closely an exponential law in view of the small difference in α calculated for various times in Tables I and III and the impressive linearity in the points on Figure 6. The calculated values of K differ somewhat in the three cases but they are of the same order of magnitude as the values deduced by Haurwitz (4) who quotes 4×10^7 gm/cm/sec for the exchange coefficient derived from the transport of heat across parallels of latitude. To obtain the corresponding figure for the eddy viscosity we have to divide this value by the density, i.e., by 9×10^{-4} gm/cm^3 at 700 mb and 7×10^{-4} gm/cm^3 at 500 mb, so obtaining $4 \times 10^{+10}$ cm^2/sec for 700 mb and $6 \times 10^{+10}$ cm^2/sec for 500 mb. Other values of the exchange coefficient have been derived by Lettau (5) which vary with latitude from 4.4×10^7 gm/cm/sec in latitude 35° to a maximum of 8.2×10^7 gm/cm/sec in latitude 50° to 55° (these correspond to $K = 5 \times 10^{10}$ and 9×10^{10} cm^2/sec respectively).

This then gives some grounds for believing that the identification of K derived from trajectories with the coefficient of eddy viscosity for lateral mixing is correct.

In Table IV, values of α and K are set out which have been derived in three independent ways. First in the columns marked A, they have been derived by the direct calculation of correlation coefficients from the velocities at various intervals of time. These coefficients given in Tables II and III have been fitted to an exponential from which α has been derived and hence K.

Secondly in the columns marked B, the values of S/σ_r have been calculated from the data in Appendix I for various values of t up to 36 hours for the trajectories in each group at 300, 500 and 700 mb (18 hours for the surface

/trajectories).

trajectories). These have been plotted on diagrams such as Figure 7 and the value of α was chosen for which the S/σ_T gave the best fit, and again K was appropriately calculated. Figure 8 shows some specimens of the fits. Thirdly in the columns marked C, the value of K has been calculated from the data in Appendix I by the use of equation (21). In these calculations t_1 was chosen to be 36 hours in the upper air groups (and 18 hours in the surface trajectories), t_2 was chosen appropriate to the biggest value of t available, (72 hours in many cases).

Thus there were two, or in some cases three, estimates of K available, and a mean value of these was taken, and compared with the corresponding standard vector deviation (σ). The correlation coefficient of K and σ was found to be 0.85, but the correlation between σ and α was small.

The regression equation of K on σ is

$$K = 7.8 \times 10^7 \sigma - 10^{10} \qquad (24)$$

where K is measured in cm^2/sec and σ in cm/sec. Figure 9 shows by dot diagram the distribution of the results of groups of trajectories. In the diagram, the large filled-in circles are used to represent initiation periods of two or three months, and three crosses indicate the mean values of K deduced from Tables I, II and III. It may be noted that if the two groups measured at 300 mb July, 1947 and February, 1948 were excluded, the correlation coefficient would be even higher at 0.883 leading to a regression equation

$$K = 9.7 \times 10^7 \sigma - 2.75 \times 10^{10} \qquad (25)$$

The expression of S as a power of the distance travelled

Contemporaneously with the preceding analysis work was done on fitting the data contained in the appendix to a power law of the form

$$S = \frac{C}{\sqrt{2}} (x)^{1-\frac{n}{2}} = \frac{C}{\sqrt{2}} (x)^{a} \qquad (26)$$

where S, as before, is the standard vector deviation of the end points of the trajectories, x the mean vector distance from the point of initiation and C and n are arbitrary constants.* Regression equations were formed of log S on log x for various time intervals up to 24 or 36 hours. This was done in the 27 cases set out in Appendix I. The correlation coefficients were very high, being 0.94 or over in all cases and 0.99 or over in 25 out of the 27, but not too much weight can be attached to such values when the logarithms of numbers are being compared. The value of a varied from 0.53 to 1.21 with an average value of 0.870. The values of C varied, of course, with the unit used, but also with the period of initiation three months, one month, a fortnight or 27 hours as the case might be; more will be said of this below.

On quite a different scale there are some rather similar data available made from series of smoke puffs fired from aircraft above a camera obscura, in a manner described by Durst (6). These puffs were plotted in the camera obscura with trajectories lasting a minute or a minute and a half; the period of initiation was generally $\frac{3}{4}$ hour to an hour. From these trajectories a plume could be made up in just the same way as is shown in Figure 1 and standard vector deviations of the end points of these trajectories were found after various time intervals. Ten such families of trajectories were worked up; the mean value of a was 0.926, the mean value of C was 0.15 in kilometre units and 0.21 in metre units.

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* Expression (26) is of the same form as that which Sutton derives from (16). His derivation cannot however be appealed to here as it would require $K/\sigma^2 t$ to be negligible and that is not the case over the range of t concerned in this paper. Hence (26) must be regarded as a purely empirical formula.

A further collection of wind observations made by smoke puffs at various heights has been discussed by Jones (7). In that case the trajectories were initiated over half-hourly periods and the value of C found when units are in centimetres was 0.275 $cm^{\frac{1}{2}}$. The value of the index, α, is not available, however, if we assume arbitrarily that the index is 0.875, the value of C in metre units is 0.16 and in kilometre units is 0.06.

The American constant level balloons referred to earlier were not suitable to provide data for working up in this manner but Moore, Smith and Gealswick (8) have discussed some cases in which two or three balloons have been launched at the same time from neighbouring sites. From photographs in the Journal of Meteorology it would seem that the initial distance apart of the balloons was of the order of 100 feet. This would correspond to a time interval of release of no more than 30 seconds even in a very light wind. In three cases two balloons were released together and in one case three balloons, which gave in all six pairs of balloons. The distance apart of the pairs after they had travelled for a long time were as follows:

Table V. Distance apart of constant level balloons

Number of balloons	Distance of flight		Distance balloons apart		Distance apart "corrected" to 1200 miles (1930 km)
	statute miles	km	statute miles	km	
2	850	1360	20	32	43 km
2	775	1250	13	21	31 km
3	1200	1930	1, 5, 5	$1\frac{1}{2}$, 8, 8	$1\frac{1}{2}$, 8, 8 km
2	1640	2630	18	29	22 km

Since the distances of the flights (l) were not the same a "correction" was made to the separation of the balloons by the formula -

$$\text{corrected separation} = \text{observed separation} \times \left(\frac{1930}{l}\right)^{0.875}$$

The root mean square of these "corrected" distances is 24 km after travel of 1930 km and the value of the vector standard deviation of a cluster of balloons which were separated by these distances would be $24/\sqrt{2}$ km or 17 km. If we assume that this standard deviation is $C/\sqrt{2}$ times the distance travelled raised to the power of 0.875, we can get a value of C from the equation

$$\log \frac{C}{\sqrt{2}} = -0.875 \log (1930) + \log 17 \qquad (27)$$

When the units are kilometres, C is 0.032 (or 0.018 for 100 kilometre units).

In addition Sutton (9) made an estimate of the value of C from the dispersion of balloons described by Richardson (10). They were released over about three hours and the value of C was 0.92 $cm^{\frac{1}{2}}$. Sutton also made an estimate of C of 0.30 $cm^{\frac{1}{2}}$ from his own observations of smoke over a grass lawn, the observations lasting about 3 minutes. If the value of α is 0.875 these amounts are equivalent to 0.52 in metre units and 0.22 in kilometre units in the case of Richardson's balloons and 0.17 in metre units and 0.07 in kilometre units in the case of Sutton's observations.

Summary of comparisons of C and α and T

The values of C derived from all the observations mentioned above have been collected together in Table VI and classified according to the period of initiation T.

/Table IV

Table VI. Mean values of a and C arranged according to the duration of the "period of initiation" (T)

Type of observation	Period of initiation (T)	Duration of trajectories	Number of cases	Mean value of a	Value of C 100 km	km	m
Geostrophic trajectories	3 months	36 hours	2	0.865	3.08	7.67	-
" "	2 months	36 hours	1	0.815	2.35	5.40	-
" "	1 month	36 hours	8	0.844	(3.07)	(8.36)	-
" "	15-28 days	36 hours	8	0.909	(1.82)	(2.23)	-
" "	27 hours	24 hours	8	0.848	(0.63)	(1.43)	-
Richardson's balloons	5 hours	-	-	-	-	0.22	0.52
Smoke puffs	$\frac{3}{4}$ hour	1 minute	10	0.926	-	(0.12)	(0.21)
Smoke puffs	$\frac{1}{2}$ hour	-	-	-	-	0.06	0.16
Sutton's smoke	3 minutes	-	-	0.875		0.07	0.17
Constant level balloons	$\frac{1}{2}$ minute	-	6	-	0.02	0.03	-

The modal values of C are put in brackets when they can be derived. The modal value of a for all the geostrophic trajectories combined is 0.878 and for the 10 smoke puffs is 0.95.

These have been compared by plotting the logarithm of "the period of initiation" (T) against the logarithm of the mean value of C in kilometre units. The result is shown in Figure 10. The correlation coefficient between $\log T$ and $\log C$ is 0.973 and the regression equation is

$$\log T = 1.99 \log C + 5.07$$

which gives $$C = 0.003\ T^{\frac{1}{2}} \qquad (28)$$

where C is expressed as $km^{\frac{1}{8}}$ and T in seconds

or $$C = 0.18\ T^{\frac{1}{2}} \qquad (29)$$

where T is expressed in hours with the qualification that it is greater than 0. It has been pointed out to us by Sir Graham Sutton that this result has been obtained theoretically by E. Inoue (15) who has shown that the diffusion angle of a smoke plume near its source increases in proportion to the square root of the time of averaging, or in our notation S at a constant distance from the source is nearly proportional to $T^{\frac{1}{2}}$.

The diffusion laws in relation to time scale

From this work it would seem that there is a remarkable agreement between the values of K (the coefficient of large scale horizontal diffusion) which we have found and that derived by other workers by quite different methods. The inference is that the Lagrangian correlation coefficients follow an exponential law. However, the empirical relation between C and T set out above would seem to support the hypothesis that the power law applies to this type of diffusion.

We believe that both are indeed correct and that the scheme below sets out the modes of diffusion.

/Eddies

$t=0$ t^* t_2 $\longrightarrow \infty$

	Eddies small in scale, effective size increasing with t		Diffusion controlled by large eddies, size distribution constant	
$r(t)$	near unity	given by power law	given by exponential law	effectively zero
Diffusion	$\sigma_T t$	$\frac{C}{\sqrt{2}}(\bar{u}t)^a$	$\frac{\sqrt{2}\,\sigma_T}{\alpha}\sqrt{\alpha t - 1 + e^{-\alpha t}}$	$P\sigma_T\sqrt{t}$

In this t is time along the trajectory, σ_T is the standard deviation of wind in the appropriate period of initiation, T, $\bar{u}$ is the mean velocity (so that $\bar{u}t$ is x) and P is a constant. It may be noted that C is a function of T and σ_T is also a function of T as can be seen in Appendix II, therefore C must itself be a function of σ_T.

It is perhaps to be expected that the value of t^* should not be precise, and that the power law and exponential forms should overlap over a considerable range of t, and this is indeed supported by what we have found from the geostrophic trajectories. We should expect the magnitude of t^* would vary from one experiment to another and to be intimately connected with the period of initiation.

Application of results to large scale diffusion in two dimensions

Instantaneous emission. In this case a 'cloud' is produced instantaneously at the source. The centre of gravity of the cloud travels along a wind trajectory, meanwhile the cloud itself spreads out (relative to its c.g.) by small scale diffusion. The examination of (geostrophic) trajectories through the source shows the area within which the centre of gravity of the cloud will have arrived after time t from the moment of generation. The most probable position will be that point arrived at by the mean trajectory (the trajectory followed by the mean wind) through the source (e.g., the appropriate monthly mean wind). If the distribution of the trajectory end-points and their standard deviation is known, the probability of the centre of gravity of the cloud arriving at any particular point at time t can be set down by assuming that the trajectories are distributed according to the exponential law. We have not been able to assemble any data on the growth of smoke clouds which would show the values of the constants in the exponential formula which are applicable to such short "periods of initiation" as are necessary for following a single cloud, if indeed the exponential law holds down to that range. Instead we have to fall back on empirical power law data which we have given in the second part of this paper. The growth of the cloud while it moves along the trajectory will take place according to local (small scale) diffusion; an approximation to the size of the cloud can be obtained by the assumption of a spreading by a power law. For this we need to know the "time of initiation" (T).

If we suppose the cloud from an explosion to be represented by a circle diameter A.B. (Figure 11) after the explosive effects are completed, which is embedded in a general wind of velocity V, the point A will be near B at a time AB/V, and we can expect the growth of the cloud after time t, represented by the difference between $A'B'$ and AB to be dependent on AB/V in the same way as the divergence of trajectories is dependent on the time of initiation.

From equation (29) we can expect then that the value of C appropriate to the diffusion of material from an explosion cloud will be given by $0.18\left(\frac{AB}{V}\right)^{\frac{1}{2}}$ if AB is measured in nautical miles and V is measured in knots.

If we use this value of C and a value of a of 0.875 it is possible to calculate the cross-wind dimensions of the cloud at various distances down wind. Suppose for instance that the wind in which the cloud is travelling is 15 kt and the initial diameter after the explosive effects have subsided is $1\frac{1}{4}$ miles then

C expressed in kilometre units is 0.052 $km^{\frac{1}{3}}$. After 12 hours the cloud would have travelled 180 miles (or 330 km) and from equation (26) S is 5.83 km or $3\frac{1}{2}$ n. miles. We can then suppose that this diffusion has taken place at each end of the diameter $C'D$ in Figure 11 and in consequence the lateral diameter of the cloud will be of the order of 9 or 10 n. miles after 12 hours.

It will be noticed that in this example it has been assumed that there is no shear in the wind across the direction of travel of the cloud. Any such shear would tend to increase the spread of the cloud but that subject need not be pursued here.

Continuous emission. In this case only a point source need be considered. The cloud of particles is emitted as a continuous plume which broadens by small scale diffusion as it gets further and further (in time) from the source. First, however, we simply ignore the broadening of the plumes and concentrate on the distribution which results from the continuous variation of the trajectories with time. The distribution of the superposed end-points of trajectories emitted over a period may as a first approximation be assumed to follow the normal (circular) law. From this it follows that the probability of an end-point falling within an elemental area $dx\,dy$ is

$$\frac{1}{\pi S^2}\exp\left\{-\frac{x^2+y^2}{S^2}\right\}dx\,dy \qquad (30)$$

where S is the standard deviation of the distance of the end-points from the centre of mass and x, y are rectangular co-ordinates with origin at the centre of mass. The x-axis is taken in the positive direction of the mean trajectory. When referred to the source as origin, x must be replaced by $(s-\bar{u}t)$ where s is distance along the mean trajectory and $\bar{u}$ the constant (or average) velocity along it.

In the case of continuous emission the successive trajectories through the source may be considered as lying within a gradually broadening "plume" and we have to consider the concentration of material across the plume (i.e., normal to the mean trajectory) at a distance s from the source. This concentration is obtained by integrating (30) over all values of t. Making the substitution for x, we obtain an expression for the concentration of diffusing substance,

$$\chi = \int_{-\infty}^{\infty}\frac{Q}{\pi S^2}\exp\left\{-\frac{(s-\bar{u}t)^2+y^2}{S^2}\right\}dt \qquad (31)$$

where Q is the rate of emission (units of mass per unit time) and S, the scatter, is dependent on the standard deviation of the wind appropriate to the "time of initiation" and also the coefficient of horizontal diffusion. This expression gives the concentration per unit length across the mean trajectory at a distance s along it, i.e., it gives the concentration along the y-axis. The integration cannot, of course, be carried out explicitly but we notice that most of the matter comprising the concentration (31) will have left the source at about the time t previously, and for matter leaving much before or after that time the contribution to χ diminishes rapidly because of the exponential factor. Hence an approximation to (31) is obtained by regarding S as appreciably constant over a small range of t centred at $s/\bar{u}$. Thus approximately;

$$\chi = \frac{Q}{\pi S^2}\exp\left(-\frac{y^2}{S^2}\right)\int_{-\infty}^{\infty}\exp\left\{-\frac{(s-\bar{u}t)^2}{S^2}\right\}dt \qquad (32)$$

$$= \frac{Q}{\pi^{\frac{1}{2}} S\bar{u}}\exp\left(-\frac{y^2}{S^2}\right) \qquad (33)$$

in which S may now be regarded as a function of time or of distance, in either case measured along the mean trajectory through the source to the region considered.

If the broadening of the plume by small scale diffusion is taken into account, it may be shown that (32) remains a first approximation since the "coefficient of diffusion" appropriate to the trajectories is several magnitudes greater than the coefficient of small-scale diffusion.

Moreover S is dependent on the period of initiation in precisely the same way as has been discussed for the geostrophic trajectories and hence we can see how the plume gradually broadens with time owing to S increasing.

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Table II. Values of $T(t)$ derived from geostrophic trajectories

Date	Height	Number of trajectories	Standard deviation (kt)	Time interval 6h		12h		18h		24h		30h		36h		42h		48h	
			S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W
1-20 Feb., 1948	700 mb	20	19.5 21.5 29.0	0.696 0.797 0.745(240)		0.539 0.513 0.517(200)		0.402 0.487 0.437(160)		0.366 0.522 0.428(120)		0.326 0.435 0.364(80)		0.262 0.346 0.299(40)		- -		- -	
17 Ap. - 4 May, 1950	700 mb	16	11.1 12.3 16.5	0.723 0.779 0.777(224)		0.577 0.628 0.609(192)		0.369 0.440 0.409(160)		0.262 0.350 0.314(128)		0.100 0.282 0.210(96)		0.076 0.295 0.210(64)		0.020 0.386 0.246(32)			
5-20 May, 1950	700 mb	16	6.3 6.9 9.4	0.731 0.700 0.715(224)		0.709 0.520 0.560(192)		0.500 0.457 0.477(160)		0.428 0.420 0.423(128)		0.271 0.438 0.362(96)		0.302 0.381 0.345(64)		0.140 0.183 0.162(32)			
1-15 June, 1950	700 mb	15	6.3 9.4 11.3	0.666 0.828 0.785(210)		0.463 0.750 0.663(180)		0.326 0.607 0.512(150)		0.304 0.505 0.443(120)		0.232 0.500 0.416(90)		0.262 0.337 0.310(60)		0.276 0.139 0.377(30)			
Oct., 1949	500 mb	30	21.2 18.1 27.9	- -		0.600 0.408 0.490(120)		- -		0.356 0.230 0.275(90)		- -		-0.013 0.148 0.057(60)		- -		-0.422 0.109 -0.150(30)	
Nov., 1949	500 mb	30	13.3 18.0 22.4	- -		0.468 0.784 0.671(120)		- -		0.257 0.641 0.499(90)		- -		0.154 0.392 0.293(60)		- -		-0.057 0.156 0.055(30)	
April, 1948	700 mb	15	11.2 13.7 17.7	- -		0.560 0.651 0.613(60)		- -		0.219 0.472 0.369(45)		- -		-0.115 0.284 0.095(30)		- -		-0.582 0.249 -0.075(15)	
May, 1948	700 mb	16	11.4 11.8 16.4	- -		0.615 0.607 0.610(72)		- -		0.221 0.238 0.229(48)		- -		0.079 0.173 0.126(32)		- -		-0.009 0.080 0.037(16)	
June, 1948	700 mb	15	11.6 10.0 13.2	- -		0.723 0.737 0.725(60)		- -		0.540 0.524 0.528(45)		- -		0.336 0.206 0.262(30)		- -		0.535 0.195 0.328(15)	
1-15 Nov., 1947	700 mb	30	18.4 14.4 23.5	- -		0.617 0.653 0.635(150)		0.401 0.662 0.549(120)		0.371 0.548 0.468(90)		0.427 0.452 0.430(60)		0.456 0.395 0.401(30)		- -		- -	
1-15 Aug, 1947	700 mb	30	7.5 6.0 9.6	- -		0.638 0.179 0.364(150)		0.385 0.070 0.204(120)		0.303 0.092 0.173(90)		0.295 -0.023 0.114(60)		0.330 0.013 0.130(30)		- -		- -	
5-day periods																			
Oct., 1949	500 mb	30	13.1 12.2 17.9	- -		0.336 0.168 0.251(120)		- -		0.135 0.060 0.118(90)		- -		-0.117 -0.021 -0.072(60)		- -		-0.380 0.132 -0.162(30)	
Nov., 1949	500 mb	30	10.5 12.2 16.1	- -		0.378 0.680 0.554(120)		- -		0.149 0.441 0.312(90)		- -		0.193 0.392 0.324(60)		- -		-0.006 0.315 0.207(30)	

Table IV. Values of α and K deduced from the geostrophic trajectories by various methods

Date	Height	Period of initiation	Average velocity over 36 hours		Standard vector deviation of wind (σ) (a) at point of initiation		(b) from all points up to 36 hours		Coefficient of eddy diffusivity for lateral mixing (A) derived from correlation coefficients		(B) derived from end-points of trajectories (0 to 36 hours)		(C) derived from end-points of trajectories by $K = \frac{S_2^2 - S_1^2}{2(t_2 - t_1)}$	
	mb	days	kt	m/s	kt	m/s	kt	m/s	hr^{-1}	cm^2/sec	hr^{-1}	cm^2/sec	hr^{-1}	cm^2/sec
Ap. to June, 1948	700	91	7	3.6	18	8.9					0.05	6.3×10^{10}	0.04	8.0×10^{10}
Oct. to Dec., 1949	500	92	21	10.3	28	14.4					0.05	14.8×10^{10}	0.07	10.5×10^{10}
17 Ap. to 15 June, 1950	700	47	9	4.6	14	7.2					0.05	9.3×10^{10}	0.04	5.0×10^{10}
April, 1948	700	30	14	7.0	21	10.4	18	9.1	0.049	6.1×10^{10}	0.07	4.6×10^{10}	0.05	7.0×10^{10}
May, 1948	700	31	6	3.1	15	7.7	16	8.3	0.053	4.8×10^{10}	0.03	6.1×10^{10}	0.05	4.5×10^{10}
June, 1948	700	30	3	1.3	11	5.7	13	6.8	0.030	5.5×10^{10}	0.02	5.8×10^{10}	0.02	5.5×10^{10}
October, 1949	500	30	25	12.6	33	16.9	28	14.4	0.064	11.6×10^{10}	0.03	3.4×10^{10}	0.10	10.5×10^{10}
November, 1949	500	30	14	6.9	28	14.0	22	11.5	0.032	14.8×10^{10}	0.07	8.3×10^{10}	0.06	11.5×10^{10}
December, 1949	500	31	21	10.3	21	10.5					0.06	6.6×10^{10}	0.03	12.0×10^{10}
July, 1949	700	20	8	4.2	15	7.4					0.01	2.0×10^{10}		
August, 1949	700	20	14	6.8	21	10.6					0.04	10.2×10^{10}		
February, 1948	700	20	24	12.0	24	12.1	29	14.5	0.042	17.5×10^{10}	0.06	8.7×10^{10}		
17 Ap. - 4 May, 1950	700	16	9	4.6	17	8.4	16	8.1	0.046	5.1×10^{10}	0.04	6.3×10^{10}	0.04	6.5×10^{10}
5 - 20 May, 1950	700	16	10	5.2	13	6.4	9	4.7	0.041	1.9×10^{10}	0.06	2.4×10^{10}	0.05	3.0×10^{10}
1 - 15 June, 1950	700	15	8	4.1	13	6.3	11	5.8	0.034	3.5×10^{10}	0.05	2.8×10^{10}	0.03	5.5×10^{10}
1 - 15 Aug., 1947	700	15	14	6.8	8	3.8	10	4.8	0.075	1.1×10^{10}	0.01	5.2×10^{10}	0.06	3.9×10^{10}
1 - 15 Nov., 1947	700	15	17	8.5	16	7.9	23	11.7	0.031	15.7×10^{10}	0.02	11.4×10^{10}	0.04	6.0×10^{10}
July, 1947	300	30	22	11.0	29	19.6					0.10	13.9×10^{10}	0.14	9.8×10^{10}
February, 1948	300	28	26	13.0	45	22.9					0.15	12.1×10^{10}	0.15	12.5×10^{10}
October, 1949	500	5					18	9.1	0.070	4.4×10^{10}				
November, 1949	500	5					16	8.3	0.043	5.8×10^{10}				
		hours	over 24 hours								(0 to 18 hours)			
21 - 22 Sept., 1943	Surface	27	16	7.8	10	5.0	11	5.8			0.15	8×10^9	0.09	10×10^9
25 - 26 Sept., 1943	"	27	14	7.1	8	4.1	8	3.9			0.10	5×10^9	0.61	1×10^9
3 - 4 March, 1944	"	27	18	9.2	6	2.9	9	4.4			0.20	3×10^9	0.04	7×10^9
6 - 7 March, 1944	"	27	11	5.7	4	1.9	7	3.5			0.35	1×10^9	0.07	2×10^9
16 - 17 June, 1944	"	27	13	6.4	14	7.0	13	6.6			0.10	15×10^9	0.13	14×10^9
22 - 23 June, 1944	"	27	8	4.0	5	2.6	7	3.5			0.50	1×10^9	0.24	1×10^9
12 - 13 Dec., 1946	"	27	17	6.8	3	1.6	5	2.5			0.75	0.3×10^9	0.09	1×10^9
16 - 17 Dec., 1946	"	27	14	6.8	5	2.4	9	4.4			0.30	2×10^9	0.03	7×10^9

Appendix I

Standard vector deviations of end-points of trajectories after various times

A. Cases at 300 mb

Time Hours	July, 1947 (300 mb) (30 trajectories) σ_x km	σ_y km	February, 1948 (300 mb) (28 trajectories) σ_x km	σ_y km
6	483	423	258	494
12	805	780	610	800
18	970	1060	940	990
24	1160	1210	1190	1180
30	1340	1420	1490	1410
36	1420	1570	1680	1580
42	1570	1760	1900	1750
48	1730	1820	2100	1890

B. Cases at 700 mb

Time Hours	August 1-15, 1947 (30 trajectories) σ_x km	σ_y km	November 1-15, 1947 (30 trajectories) σ_x km	σ_y km
6	124	83	211	171
12	237	209	417	362
18	409	266	597	510
24	513	344	765	687
30	666	390	905	805
36	887	430	1100	950
42	1026	460	1288	1098
48	1253	521	1500	1198
54			1742	1293
60			1975	1391

C. Cases at 700 mb

Time Hours	April, 1948 (15 trajectories) σ_x km	σ_y km	May, 1948 (16 trajectories) σ_x km	σ_y km	June, 1948 (15 trajectories) σ_x km	σ_y km	April - June, 1948 (46 trajectories) σ_x km	σ_y km
6	198	225	81	166	25	123	98	192
12	392	462	161	317	51	245	193	363
18	500	603	197	464	103	352	252	482
24	575	727	260	617	129	458	297	650
36	915	922	405	838	167	644	462	888
48	1255	1162	489	985	240	933	621	1142
60	1685	1435	563	1208	312	1168	801	1436
72	2045	1630	710	1357	423	1343	1014	1686

D. Cases at 500 mb

Time Hours	October, 1949 (31 trajectories) σ_x km	σ_y km	November, 1949 (30 trajectories) σ_x km	σ_y km	December, 1949 (31 trajectories) σ_x km	σ_y km	Oct.-Dec., 1949 (92 trajectories) σ_x km	σ_y km
6	197	364	87	302	183	226	150	310
12	442	688	218	509	372	424	330	566
18	690	1042	380	658	541	583	523	800
24	980	1337	510	890	770	744	755	1050
30	1291	1600	682	1100	1066	882	1030	1270
36	1630	1675	897	1378	1375	1028	1330	1430
42	1924	1870	1033	1562	1710	1160	1620	1630

Time Hours	October, 1949 (31 trajectories) x km	October, 1949 (31 trajectories) S km	November, 1949 (30 trajectories) x km	November, 1949 (30 trajectories) S km	December, 1949 (31 trajectories) x km	December, 1949 (31 trajectories) S km	Oct.-Dec., 1949 (92 trajectories) x km	Oct.-Dec., 1949 (92 trajectories) S km
48	2255	1973	1253	1760	2100	1285	1950	1770
54	2485	2087	1500	1915	2358	1427	2225	1910
60	2655	2155	1710	2042	2750	1598	2530	2060
66	2820	2155	1937	2157	2840	1827	2700	2110
72	-		2250	2205	3530	2035	2800	2170

E. Cases at 700 mb

Time Hours	17th Ap.-4th May, 1950 (32 trajectories) x km	17th Ap.-4th May, 1950 (32 trajectories) S km	5-20th May, 1950 (32 trajectories) x km	5-20th May, 1950 (32 trajectories) S km	1-15th June, 1950 (30 trajectories) x km	1-15th June, 1950 (30 trajectories) S km	17th Ap.-15th June, 1950 (94 trajectories) x km	17th Ap.-15th June, 1950 (94 trajectories) S km
6	99	181	96	138	81	135	89	155
12	203	339	197	240	176	226	186	274
18	317	470	328	349	262	340	302	393
24	408	604	441	440	358	432	397	501
30	523	754	565	523	447	534	510	618
36	600	947	670	576	533	628	600	743
42	717	1067	805	682	621	747	716	853
48	837	1174	885	756	722	856	815	950
54	972	1325	975	832	810	968	912	1064
60	1127	1415	1110	914	867	1049	1046	1157
66	1244	1520	1225	995	1130	1220	1195	1270
72	1389	1613	1387	965	1410	1358	1388	1348

F. Cases at 700 mb

Time Hours	1-20th February, 1948 (40 trajectories) x km	1-20th February, 1948 (40 trajectories) S km	July, 1949 (20 trajectories) x km	July, 1949 (20 trajectories) S km	August, 1949 (20 trajectories) x km	August, 1949 (20 trajectories) S km
6	375	261	134	160	171	230
12	675	490	223	317	287	450
18	918	629	329	469	453	631
24	1113	807	405	615	594	827
30	1327	987	480	759	749	973
36	1549	1160	547	887	887	1137

G. Surface cases with time of initiation 27 hours

Time Hours	21/22nd Sept., 1943 (18 trajectories) x km	21/22nd Sept., 1943 (18 trajectories) S km	25/26th Sept., 1943 (18 trajectories) x km	25/26th Sept., 1943 (18 trajectories) S km	3/4th Mar., 1944 (18 trajectories) x km	3/4th Mar., 1944 (18 trajectories) S km	6/7th Mar., 1944 (18 trajectories) x km	6/7th Mar., 1944 (18 trajectories) S km
3	87	54	49	44	105	31	77	21
6	183	103	90	79	202	63	145	43
9	275	144	174	103	295	94	201	59
12	375	186	261	144	407	113	254	71
15	472	223	351	158	502	148	311	88
18	554	257	453	180	613	167	345	113
21	618	293	528	191	709	210	419	134
24	671	331	611	197	792	247	496	150

Time Hours	16/17th June, 1944 (18 trajectories) x km	S km	22/23rd June, 1944 (18 trajectories) x km	S km	12/13th Dec., 1944 (18 trajectories) x km	S km	16/17th Dec., 1944 (18 trajectories) x km	S km
3	81	75	45	28	77	17	82	26
6	168	138	88	59	151	37	165	59
9	253	194	130	79	225	48	236	81
12	321	242	167	91	300	55	300	104
15	381	283	201	89	378	61	378	143
18	447	321	244	79	452	68	453	169
21	504	365	290	81	520	80	527	195
24	553	365	343	95	587	93	585	225

Appendix II

The values of σ_τ over Southern England

The standard deviation of wind (σ_τ) taken over intervals of duration τ bears a ratio to the standard deviation over a long period (σ) depending on the value of τ.

The ratios have been set out in Tables A and B. Table A was derived from the comparison of winds at 500 mb observed during 15 years at Larkhill (or Crawley) in the months of January and July. Table B was derived from data given in Geophysical Memoir 93 (14) which enabled the ratios σ_τ to σ to be calculated.

Table A Value of σ_τ (in kt) for various values of τ

	τ	1 day	2 days	4 days	8 days	1 mth.	3 mth.	5 mth.	15 mth.
January	σ_τ	17.7	24.4	29.4	32.6	38.8	40.6	41.6	41.8
	σ_τ/σ_{15}	0.40	0.58	0.70	0.78	0.93	0.97	0.99	1.00
July	σ_τ	11.6	16.2	19.4	22.6	28.4	29.3	29.5	29.6
	σ_τ/σ_{15}	0.39	0.55	0.65	0.76	0.96	0.99	1.00	1.00

Table B Values of σ_τ/σ for various values of τ

τ	5 min.	15 min.	30 min.	1 hr.	6 hr.	12 hr.	24 hr.
σ_τ/σ	0.037	0.062	0.088	0.111	0.22	0.30	0.41

σ being the long period value of the standard vector deviation of wind which is a constant for the appropriate region, season and level.

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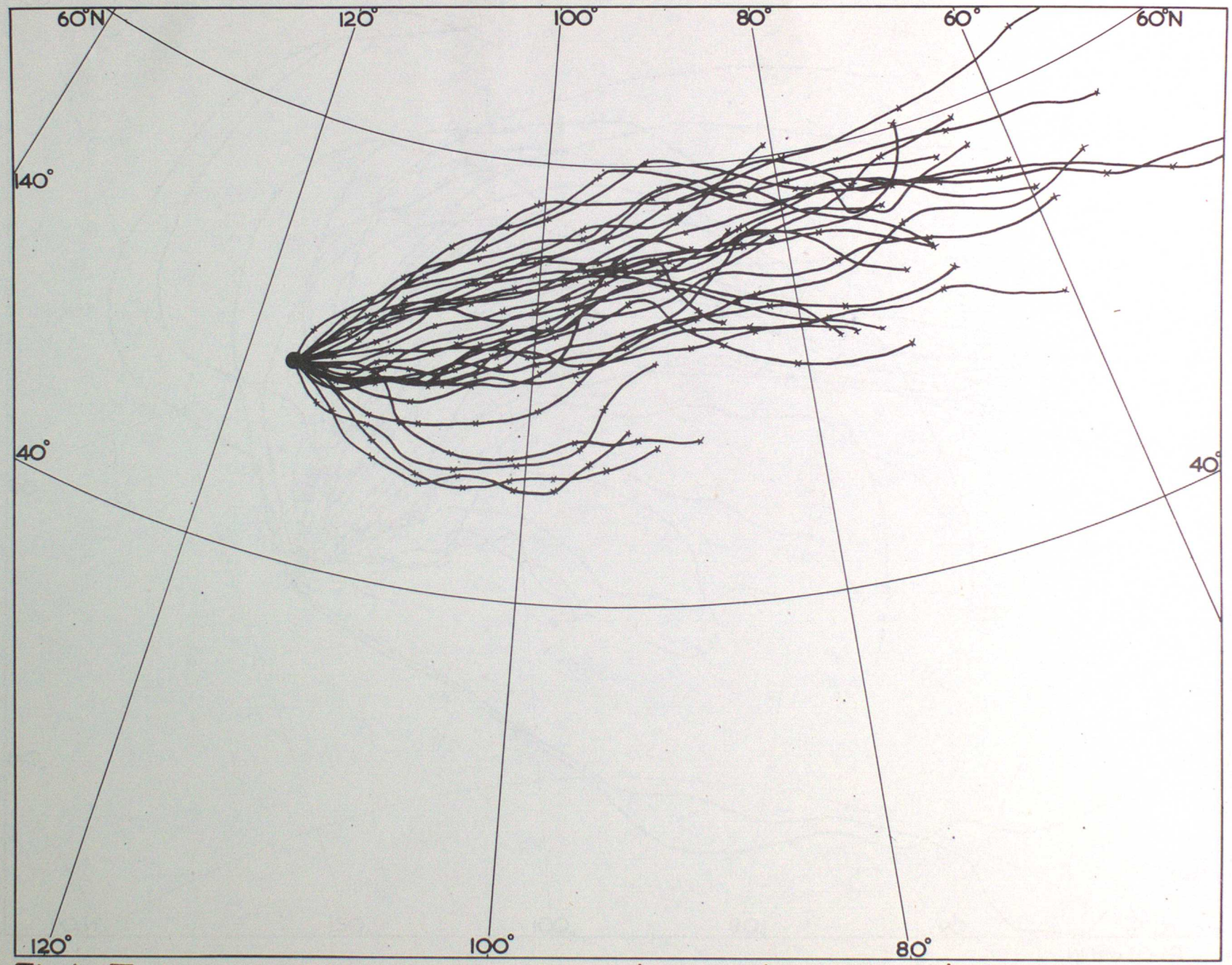

Fig. 1a Trajectories initiated twice a day and pursued for 4 days during 1-15 Aug. 1947.

M.R.P. 1058

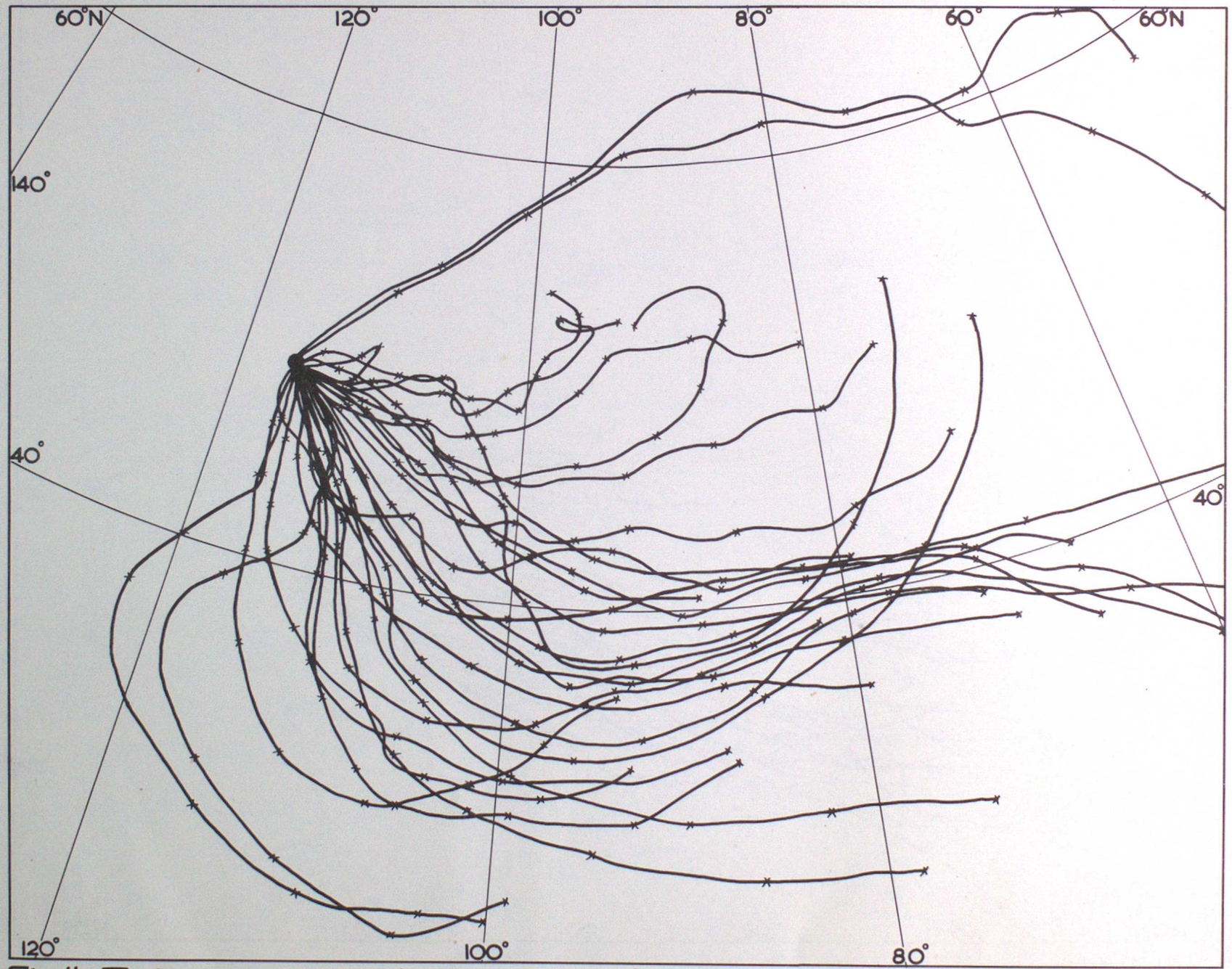

Fig. 1b. Trajectories initiated twice a day and pursued for 4 days during 1-15 Nov. 1947.

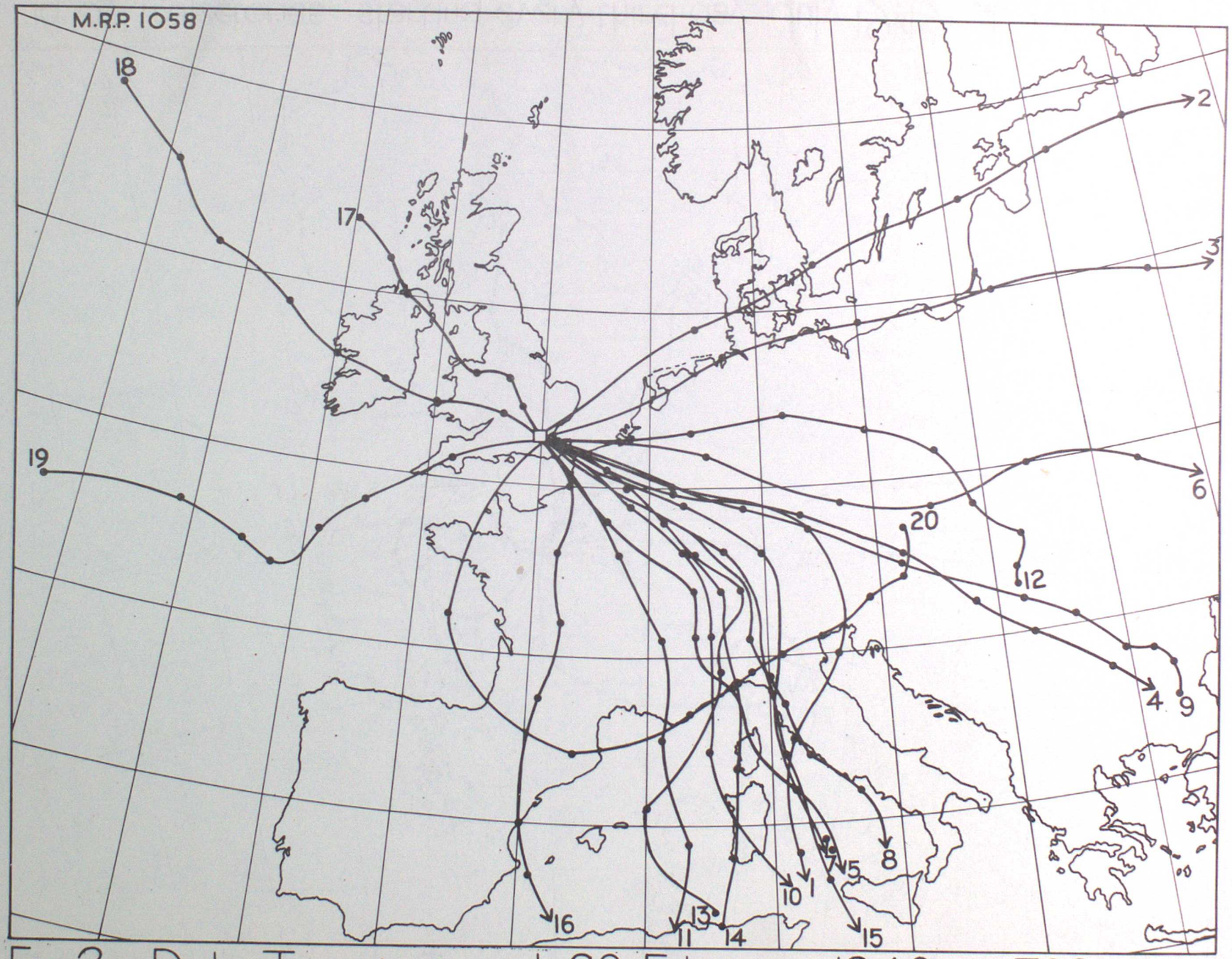

Fig. 2 Daily Trajectories. 1-20 February 1948 at 700 mb.

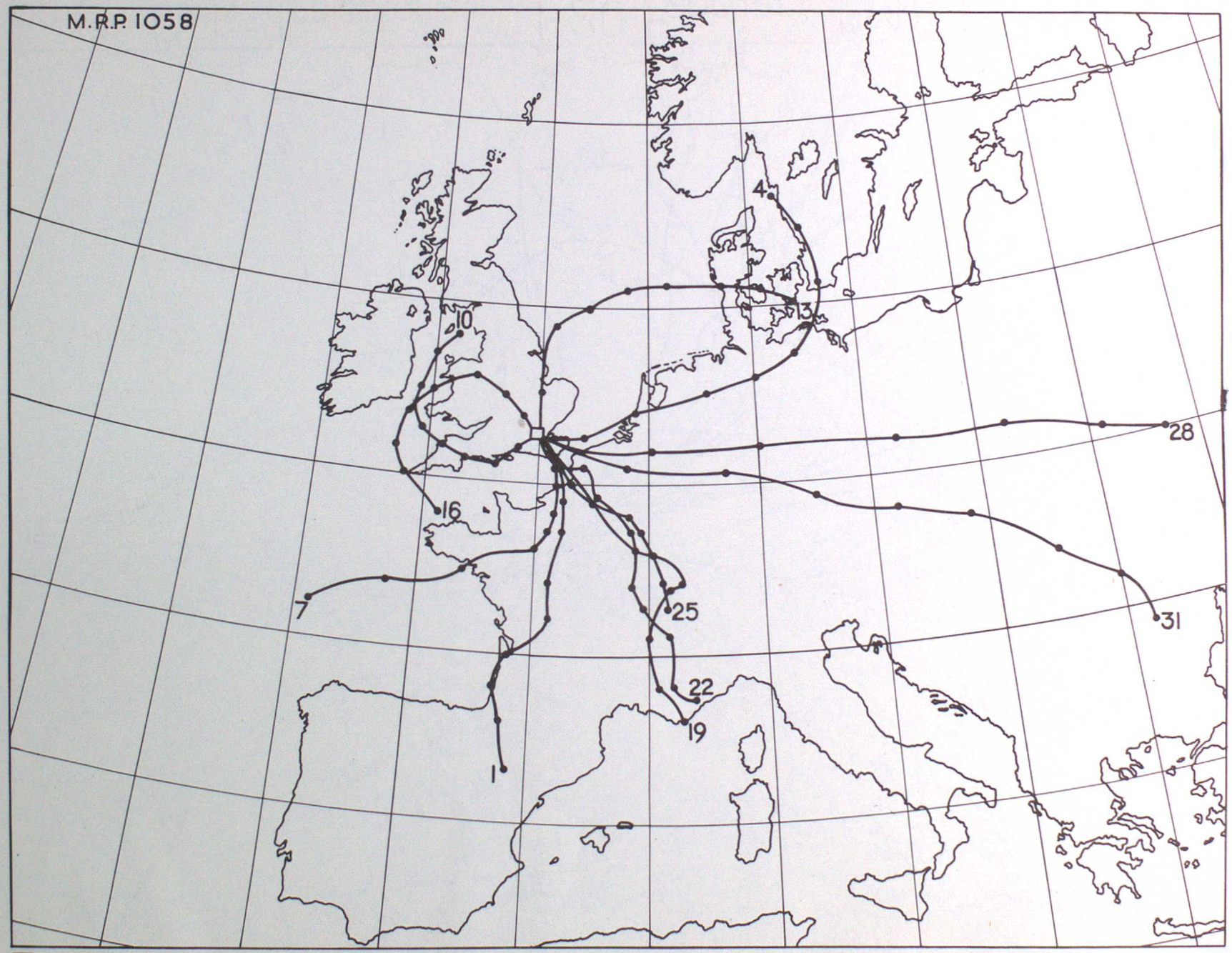

Fig. 3 Trajectories starting every third day. July 1949 at 700 mb.

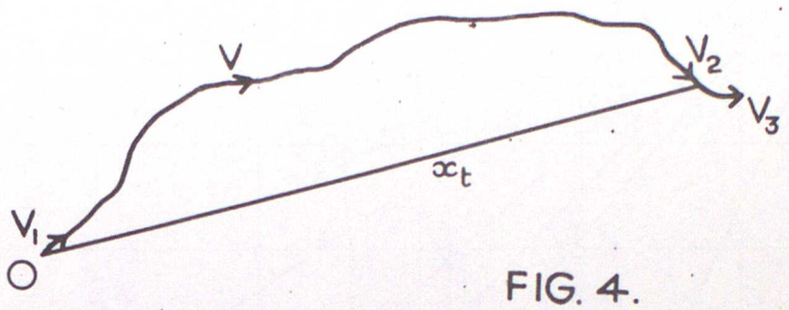

FIG. 4.

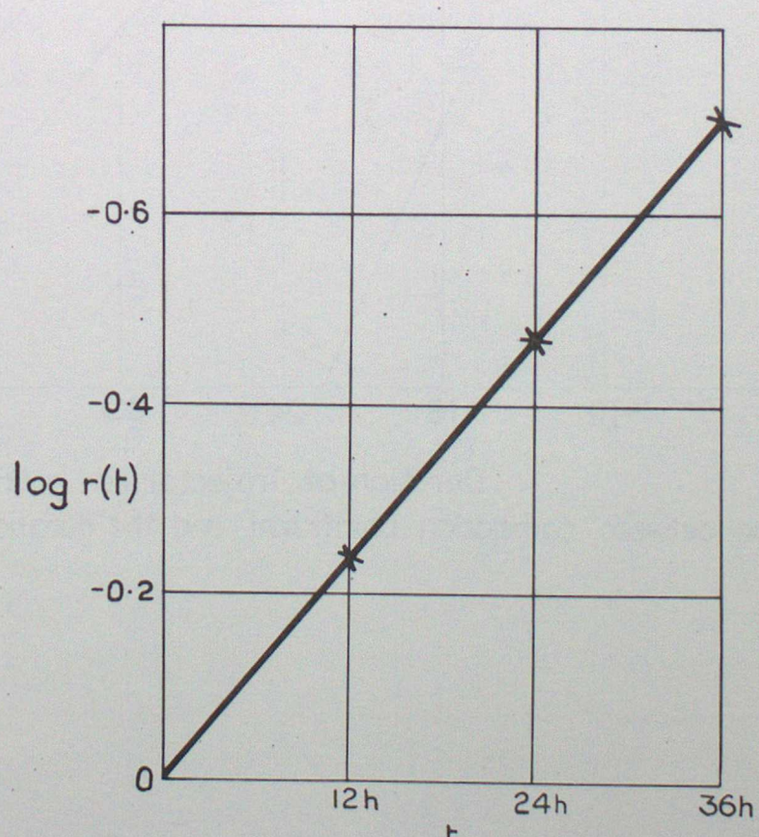

FIG 6. Relation of r(t) and t.

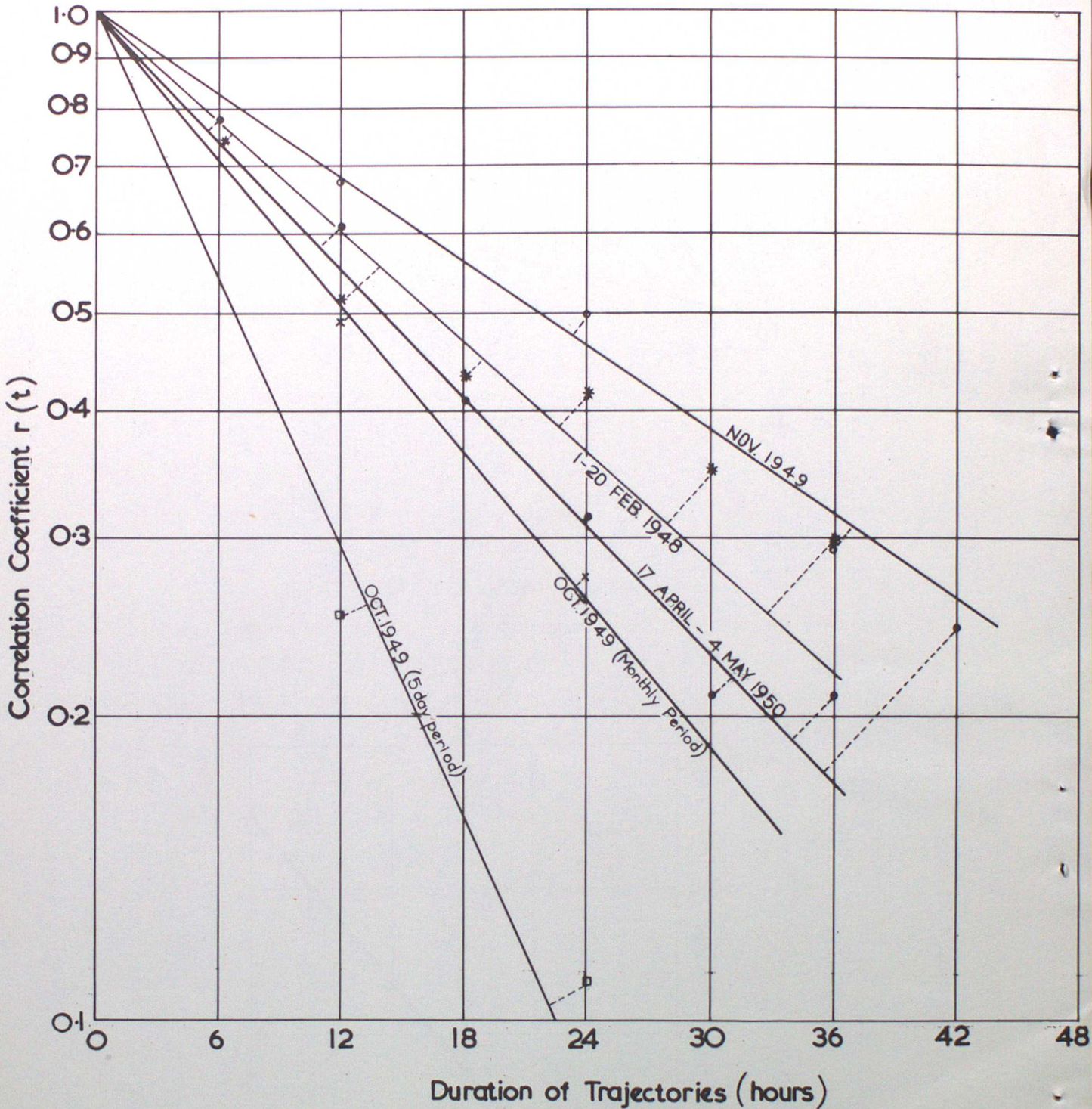

Fig. 5. Relation between correlation coefficient and the duration of trajectories.

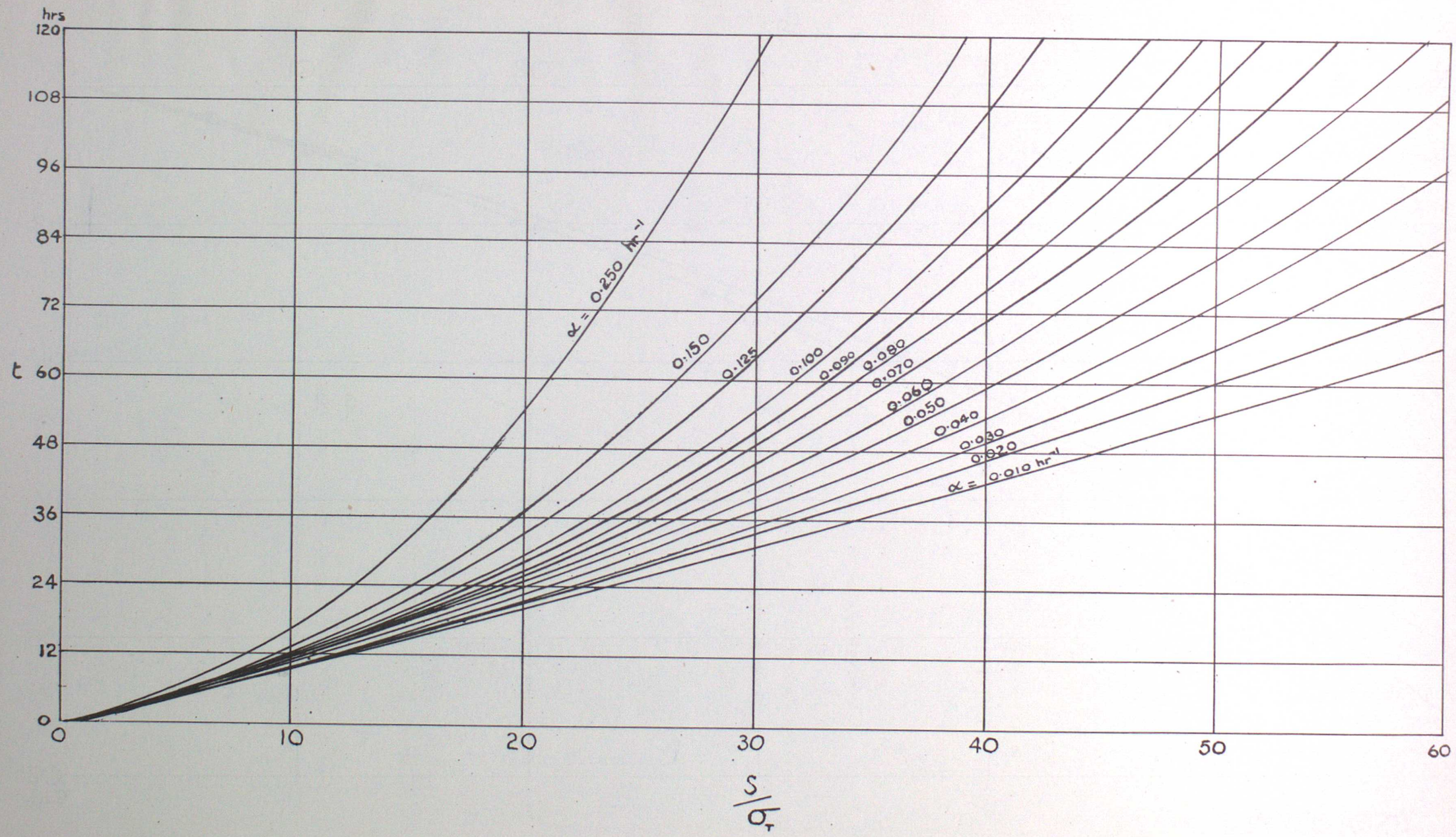

Fig. 7 Values of ratio S/σ_T for various values α and t.

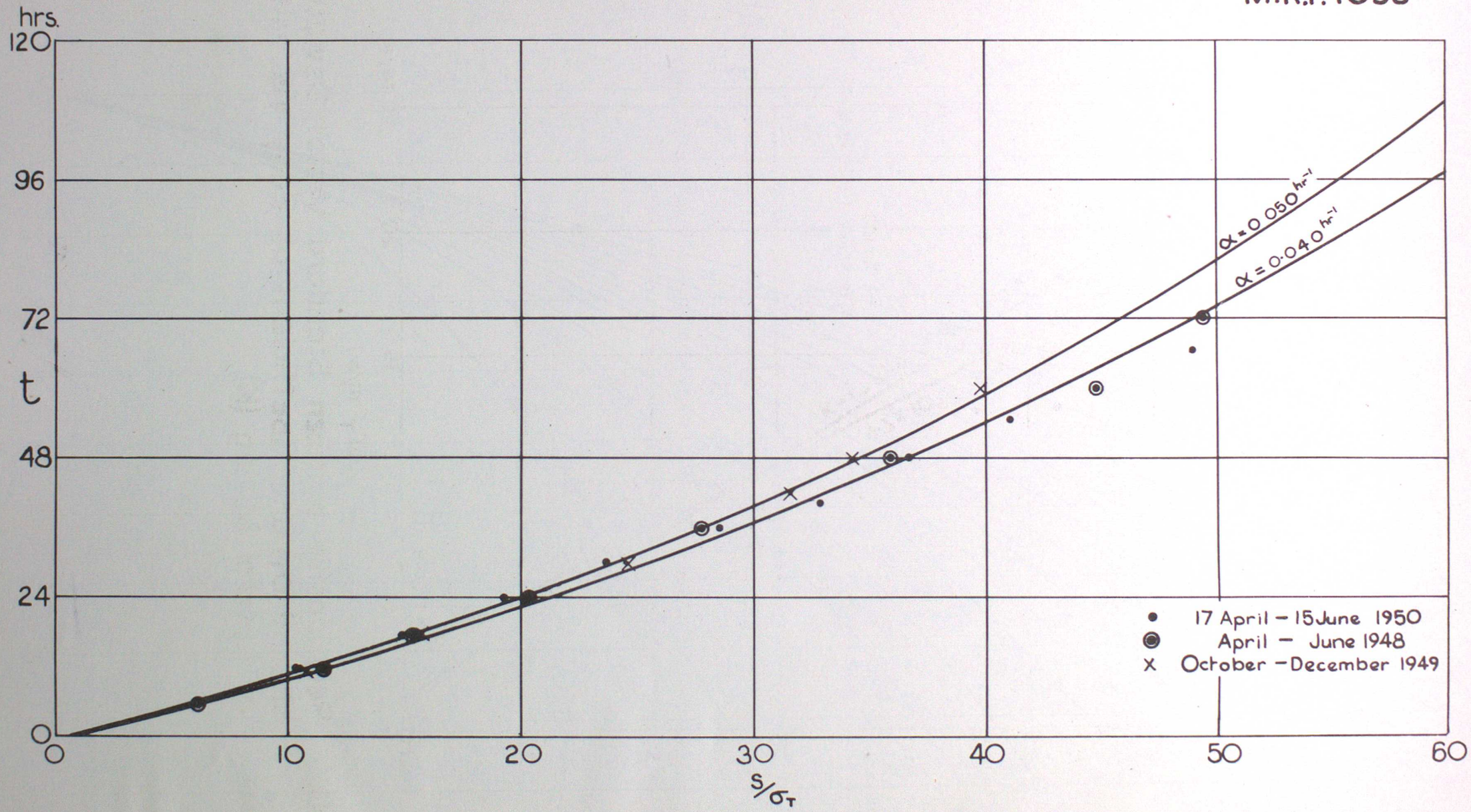

Fig 8 α obtained from Trajectories

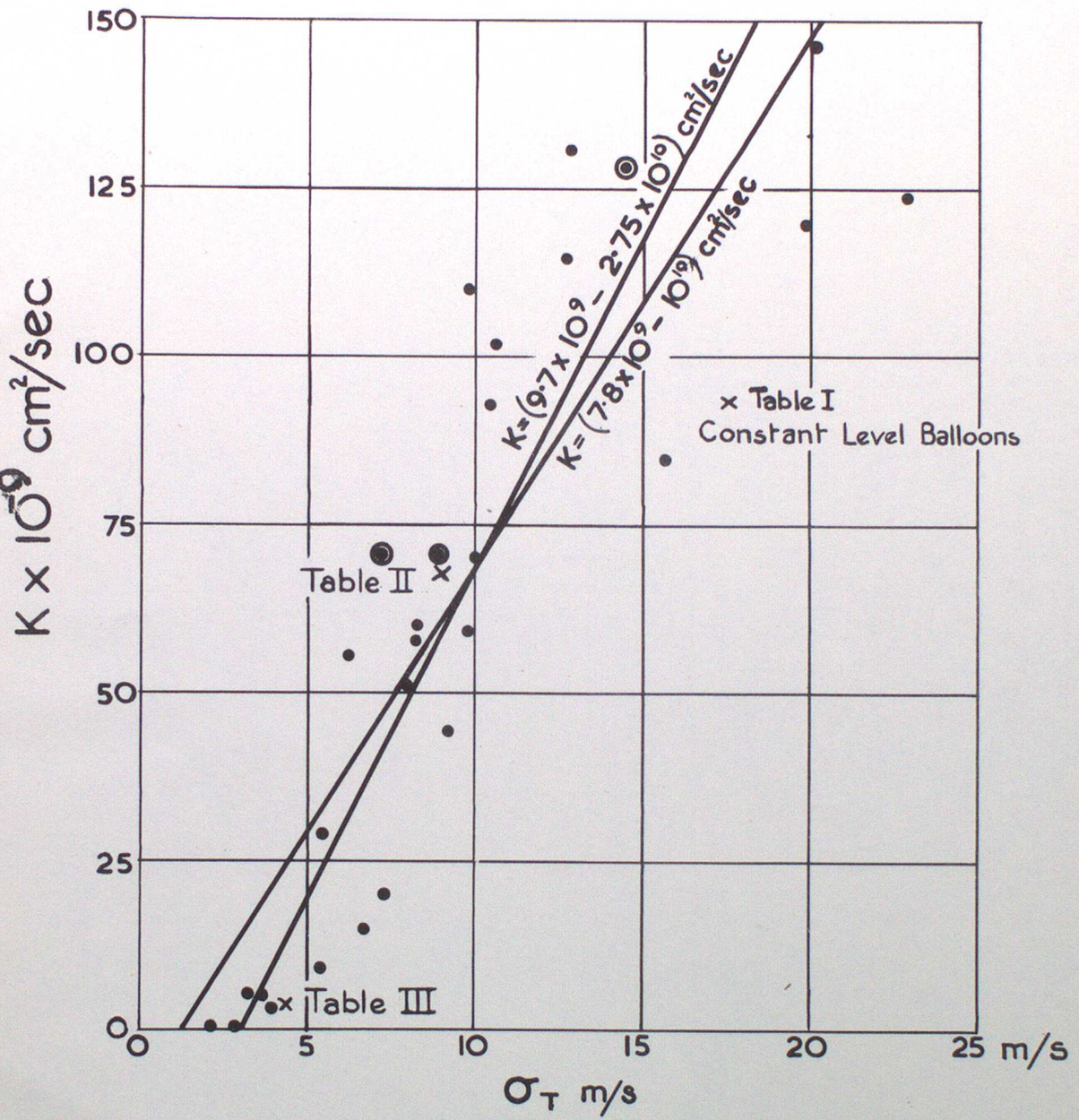

FIG. 9 Relationship between Standard Vector Deviation of wind (σ_T) and Coefficient of Eddy Viscosity for Lateral Mixing (K).

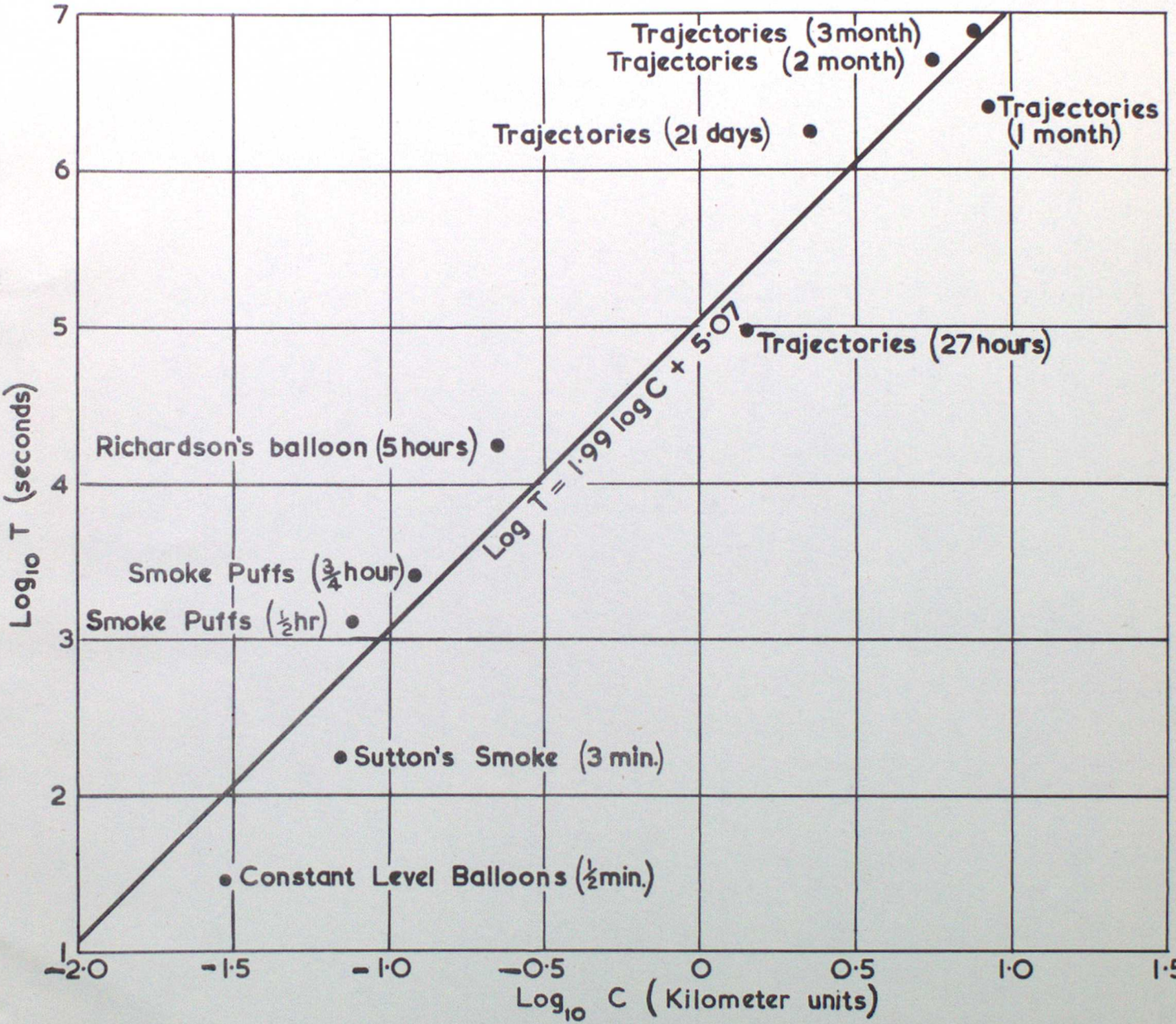
Trajectories (3 month)
Trajectories (2 month)
Trajectories (21 days)
Trajectories (1 month)
Trajectories (27 hours)
Log T = 1·99 log C + 5·07
Richardson's balloon (5 hours)
Smoke Puffs (¾ hour)
Smoke Puffs (½ hr)
Sutton's Smoke (3 min.)
Constant Level Balloons (½ min.)
Log10 T (seconds)
Log10 C (Kilometer units)
7
6
5
4
3
2
1
−2·0
−1·5
−1·0
−0·5
0
0·5
1·0
1·5

FIG. 10

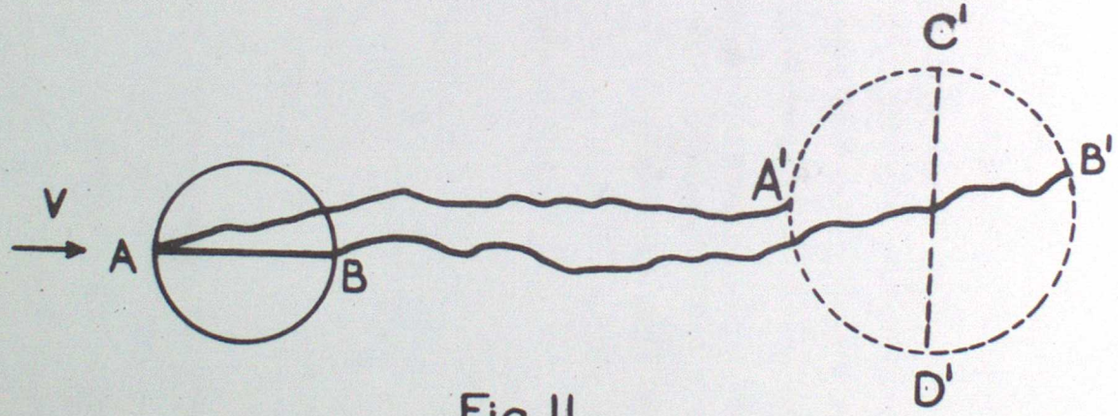

Fig 11